

REDUCING CONTRASTIVE KNOWLEDGE

Abstract

According to one form of epistemic contrastivism, due to Jonathan Schaffer, knowledge is not a binary relation between an agent and a proposition, but a ternary relation between an agent, a proposition, and a context-basing question. In a slogan: to know *is* to know the answer to a question. I argue, first, that Schaffer-style epistemic contrastivism can be semantically represented in inquisitive dynamic epistemic logic, a recent implementation of inquisitive semantics in the framework of dynamic epistemic logic; second, that within inquisitive dynamic epistemic logic, the contrastive ternary knowledge operator is reducible to the standard binary one. This shows, I argue, that knowledge is not essentially question sensitive, contrary to what the contrastivist claims. It follows that the contrastivist's response to radical skepticism is ungrounded.

1 Introduction

According to Jonathan Schaffer, knowledge is *question relative* [Schaffer, 2007b]. In terms of its logical form, knowledge is a *relation*: according to the traditional, or binary view, knowledge is a relation between an agent and some other object, usually a proposition. So according to the binary view, “a knows that p” is just the binary relation $K(a, p)$, where a stands for an agent and p stands for a proposition. According to Jonathan Schaffer, knowledge is $K(a, p, Q)$, a ternary relation between an agent, a proposition and a question. In this paper, *epistemic contrastivism* will denote the latter ternary view about knowledge.^{1,2}

In a series of papers, Jonathan Schaffer has defended and developed his contrastivist position,³ making some impact on current debates.⁴ In the remaining

¹Schaffer usually presents contrastivism as the view that knowledge is a ternary relation between an agent, a proposition and a contrast proposition q (i.e. $(K(a, p, q))$), but he takes it for granted that the two are interchangeable:

contrastive knowledge is equivalent to question-relative knowledge: $K(a, p, q)$ is equivalent to $K(a, p, Q)$, where Q is the question $?\{p, q\}$ [i.e. the question whether p or q]. [Schaffer, 2007a, p.241]

²There are other contrastive views, notably [Dretske, 1972] and [Karjalainen and Morton, 2003]. Here I focus on Schaffer's views.

³See [Schaffer, 2004], [Schaffer, 2005], [Schaffer, 2007b], [Schaffer, 2007a], [Schaffer, 2008], [Schaffer and Knobe, 2012] and [Schaffer and Szabo, 2014].

⁴See, e.g. [Rysiew, 2011] and [Stanley, 2011].

parts of this section, I will present some of Schaffer’s main arguments in favor of contrastive knowledge. Schaffer is offering a contrastive account which explains both *knowledge-that* clauses and *knowledge-wh* clauses (clauses in which the verb *to know* is followed by a *wh*-question, such as *whether*, *where*, *when*, etc.). In sections 2, I review the logical preliminaries, and in section 3 I offer a formalization of contrastive knowledge, based on inquisitive dynamic epistemic logic.⁵ I argue that the formalization succeeds in capturing Schaffer’s contrastive ideas, both for *knowledge-that* and for *knowledge-wh*. In section 4, I focus on the consequences of this formalization, which include a reduction of contrastive knowledge to binary knowledge. I argue that this reduction threatens Schaffer’s claim that contrastive knowledge is genuinely distinct from the binary conception of knowledge.

1.1 The problem of convergent knowledge

According to the received view, *knowledge-wh* is reducible to *knowledge-that* (see e.g. [Hintikka, 1975], [Lewis, 1982], [Stanley, 2011]). The reductive thesis claims that knowing-*wh* Q , where Q is some *wh*-question, is equivalent to knowing that p , where p is the true answer to the question Q . For instance, I know *whether* p iff I know *that* p or I know *that* not- p (where either p or not- p constitute the answer to the question *whether* p); I know *who* killed J. F. K iff I know *that* Oswald killed J. F. K (where “Oswald killed J. F. K” constitutes a true answer to the question “Who killed J.F.K?”).

The reductive analysis provides a simple and unifying account for *knowledge-that* and *knowledge-wh*, but it faces the *problem of convergent knowledge* [Schaffer, 2007b]. If two questions, Q_1 and Q_2 , have the same answer (i.e., they converge on the answer), then the received view predicts that an agent who has *knowledge-wh* of Q_1 must have *knowledge-wh* of Q_2 . Schaffer claims that there are simple counterexamples to this prediction.

Consider Alice, standing at her friend’s garden. She sees a small yellow bird, a goldfinch in fact, at the distance. Compare the sentences

- (1) Alice knows whether it’s a goldfinch in the garden, or a raven.
- (2) Alice knows whether it’s a goldfinch in the garden, or a canary.
- (3) Alice knows whether it’s a goldfinch in the garden, or at the neighbor’s.

The answers to all the embedded questions in (1) to (3) converge to the proposition “there is a goldfinch in the garden.” Hence, the received view is committed to the truth functional equivalence of (1) to (3). But, as Schaffer notes, the three seem to be different. Alice might be a well-respected ornithologist, making both (1) and (2) true; for (3) to be true, however, Alice has to know where does her friend’s

⁵For other formal representations of Schaffer’s contrastivism, see [Aloni et al., 2013], [Schaffer and Szabo, 2014] and [Hawke, 2016]. None of these focus on the question of the reduction of contrastive knowledge, which makes them less suitable for my purposes here.

garden ends, which is a completely different task than the task of bird identification. Alternatively, Alice might not know that much about birds (s.t. (1) is true but not (2)); however, she knows very well the exact borders of the garden (s.t. (3) is true). Let us fix the latter scenario to be the case.

Schaffer offers his contrastive account as an explanation to these examples. The binary account of the knowledge relation will represent (1) as $K(a, Q_1)$, a relation between Alice and a certain object, the question “Is it a goldfinch or a raven?” But Schaffer argues that this is *misleading*: the genuine logical structure of (1) is $K(a, Q_1, Q_1)$. What (1) is really saying is: given the question “Is it a goldfinch or a raven?”, Alice knows whether it’s a goldfinch or a raven. Similarly, what (2) is really saying is $K(a, Q_2, Q_2)$: given the question “Is it a goldfinch or a canary?”, Alice knows whether it’s goldfinch or a canary. Although Q_1 and Q_2 have the same answer, they determine different contrasts: in (1), Q_1 determines a contrast between a goldfinch and a *raven*; in (2), Q_2 determines a contrast between a goldfinch and a *canary*. The different contrasts explain why (1) and (2) are not equivalent. Q_1 and Q_2 do not have the same contrast, and identification, and so knowledge, is contrast sensitive.

The problem of convergent knowledge focuses us on knowledge-wh, but Schaffer’s claim generalizes to knowledge-that as well. Consider the sentences:

- (4) Alice knows that there is a goldfinch in the garden, given the question “Is it a goldfinch, or a raven?”
- (5) Alice knows that there is a goldfinch in the garden, given the question “Is it a goldfinch, or a canary?”

(4) and (5) are the analogous knowledge-that claims to (1) and (2), respectively. As such, (4) is intuitively judged true, while (5) judged false. In (4) and (5), the contrastive structure is clearer, as we can see the corresponding agent, proposition and question. As a three-place relation, sentences (4) and (5) can be represented as $K(\text{Alice}, p, Q_1)$ and $K(\text{Alice}, p, Q_2)$, respectively: in both sentences we have the same agent (Alice), the same known proposition (that there is a goldfinch in the garden), but different contrast-basing questions (canary as opposed to raven).

Examples (1) to (3) have supposedly shown that knowledge-wh claims require an extra argument for a contrast-basing question. If we are after a uniform account of knowledge, it is natural to require that knowledge-that takes a contrast-basing question *as well*. If ordinary knowledge-that talk does not make the contrast question explicit, it is implicitly provided by the context – so says the contrastivist. This conclusion provides us with a modified reductive thesis regarding the relation between knowledge-that and knowledge-wh: when the contrast-basing question is held fixed, knowing-wh Q is equivalent to knowing that p , where p is the true answer to the question Q . In this modified sense, sentence (1) is equivalent to (4), and (2) to (5).

Contrastivists believe that they have an answer to the external-world skeptic. Moore knows that he has hands rather than hooks (i.e. given the question “do

you have hands or hooks?”); Moore does not know that he has hands rather than having hallucinations of hands as a brain-in-a-vat (i.e. given the question “do you have hands or hallucinations of hands?”) [Schaffer, 2007b]. The skeptic’s mistake is in assuming that knowing that I have hands can be evaluated independently of contrast, i.e. that the knowledge relation is binary. The contrastivist rejects this assumption.

1.2 The reduction of contrastive knowledge

According to Schaffer, contrastive knowledge cannot be reduced to binary knowledge. After reviewing and rejecting a few strategies to reduce contrastive knowledge to the binary type, he concludes

... that contrastivism is indeed the only option—the contrast-sensitivity of knowledge ascriptions shows that knowledge is a contrastive relation. [Schaffer, 2008, p. 235]

Since the binary theorist cannot explain the apparent contrast sensitivity of knowledge ascriptions, it follows that knowledge ascriptions are contrastive and therefore that *the knowledge relation itself* is contrastive, according to Schaffer.

Instead of arguing for and against the reduction procedure of particular sentences, in this paper I would take a more general approach. As a first step, I distinguish between two kinds of reductions, **R1** and **R2**, both involve reduction of sentences to sentences. **R1** reduction reduces (sentences about) contrastive knowledge to (sentences about) binary knowledge by internalizing the contrast-basing question in the proposition known. Formally:

$$\mathbf{R1}: K(a, \varphi, Q) \Leftrightarrow K(a, \varphi^*)$$

φ^* therefore somehow combines information from the question Q and from the proposition φ known contrastively. **R2** reduction is a more general: it reduces contrastive knowledge into a sentence that possibly contains the binary knowledge relation (among other things), but not the contrastive relation. Formally put:

$$\mathbf{R2}: K(a, \varphi, Q) \Leftrightarrow \varphi^* \quad \text{where } \varphi^* \text{ does not contain contrastive knowledge.}$$

Since the expression $K(a, \varphi^*)$ is itself a sentence without contrastive knowledge, **R1** entails **R2**. The converse does not hold.

Although Schaffer does not make the distinction between the two kinds of reductions, he needs to reject both of them. Schaffer argues that *all* knowledge is contrastive, that there is no binary knowledge. According to **R2**, sentences with contrastive knowledge are reducible to sentences that possibly contain binary knowledge, but no contrastive knowledge. Suppose, with Schaffer, that all knowledge is contrastive and that there is no binary knowledge in reality. Then a true description of the world (in the form of a declarative sentence) which describes contrastive knowledge could not be reformulated as a true description of the world with only

binary knowledge. In other words, according to Schaffer’s position, **R2** *cannot* be truth preserving. Hence, **R1** should be impossible as well.

If a reduction of contrastive knowledge is shown to be possible, contrastivism, in its strong form, is in danger. In such a case, everything that can be said about knowledge can be said using binary knowledge – there will be no need to change our entire conception of knowledge as a binary relation.

In what follows I formulate contrastivism in the framework of inquisitive dynamic epistemic logic. My aim is twofold: first, to work towards a formal semantics that can explain the contrastive phenomena and explicate the informal ideas of contrastivism as an epistemological theory. Second, to investigate the question of the reduction of contrastive knowledge in a systematic manner. I show that there is a sense in which contrastive knowledge is systematically reducible to binary knowledge, which is the **R2** sense.

The next section introduces the logical tools needed to accomplish these aims.

2 Inquisitive dynamic epistemic logic

Inquisitive epistemic logic (IEL) is an extension of standard epistemic modal logic in the framework of *inquisitive semantics* [Ciardelli et al., 2015]. We will use the epistemic logic to represent knowledge and the inquisitive semantics to represent questions. We start with a recap of standard epistemic modal logic, which we then extend to the framework of inquisitive semantics.

2.1 Epistemic logic and its dynamics

The language of propositional epistemic logic is the language of propositional logic extended with the modal K_a operator. We read $K_a\phi$ as “a knows ϕ .” An *epistemic model* for a set \mathcal{P} of propositional letters is a structure $\mathcal{M} = (W, V, \sigma)$ s.t.

- W is a non-empty set of possible worlds, also referred to as the *logical space*,
- V is a valuation function $V : \mathcal{P} \rightarrow P(W)$, specifying the set of possible worlds in which any given propositional letter is true, and,
- σ is an epistemic map, $\sigma : W \rightarrow P(W)$, assigning each world w an information state $\sigma(w)$ (for a given agent).⁶

An *S5 epistemic model* is an epistemic model in which σ respects factivity and introspection:

- Factivity: for any $w \in W$, $w \in \sigma(w)$.
- (Positive and negative) Introspection: for any $w, v \in W$, if $v \in \sigma(w)$, then $\sigma(w) = \sigma(v)$.

⁶Using the σ function is equivalent to the (more familiar) use of an epistemic accessibility relation R where $\sigma(w) = \{v \in W \mid wRv\}$.

Factivity is a necessary requirement for knowledge representation, capturing the intuition that knowledge is factive. Introspection is an idealized requirement, capturing the assumption that our ideal agent knows what she knows and does not know. There are good reasons to reject this introspection assumption, but we retain it here for technical simplicity. This should be admissible, as this work will not be concerned with higher states of knowledge.

Sentences are evaluated at worlds: in particular, we demand that $K\phi$ is true at w iff it is true in the entire information state of w , i.e.

$$\mathcal{M}, w \models K\phi \Leftrightarrow \forall v, v \in \sigma(w) : \mathcal{M}, v \models \phi.$$

and the satisfaction conditions for the other Boolean connectives are straightforward (see [van Ditmarsch et al., 2007] for full details).

Epistemic logic can be extended to dynamic epistemic logic (DEL). DEL can express and represent epistemic *change*. We will briefly go over the (single agent) DEL public announcement logic (PAL). PAL can represent the agent’s epistemic change after learning true information (“announcements”, broadly conceived). For this, we add to the language the modality $\langle \phi \rangle$, s.t. $\langle \phi \rangle \psi$ is to be read as “after announcing ϕ , ψ holds.” Semantically, announcing that ϕ in model \mathcal{M} has the effect of eliminating all the not- ϕ worlds from the model. The resulting model after the announcement is then written as \mathcal{M}_ϕ , s.t. $W_\phi = W \cap \{w : w \models \phi\}$; σ and V are then restricted according to W_ϕ . We then have the truth clause

$$\mathcal{M}, w \models \langle \phi \rangle \psi \Leftrightarrow \mathcal{M}, w \models \phi \text{ and } \mathcal{M}_\phi, w \models \psi,$$

specifying that $\langle \phi \rangle \psi$ is true whenever the announced sentence ϕ is true and ψ holds in the model resulting from the announcement of ϕ (for more details regarding PAL, see [van Ditmarsch et al., 2007]).

2.2 Inquisitive epistemic logic and its dynamics

The standard possible worlds semantic framework identifies propositions with sets of possible worlds. Inquisitive semantics takes a more fine-grained approach to propositions, in which a proposition is *a set of sets of possible worlds*. So, for instance, the proposition expressing p or not- p is a set that contains the set of the p worlds and the set of the not- p worlds. Why should we identify propositions as sets of sets of possible worlds? One can think of propositions as having two kinds of contents: informative content and inquisitive content. The informative content of a proposition can be identified with the information expressed in it, and the inquisitive content with the *issue* which is raised by it [Ciardelli et al., 2015]. Inquisitive semantics will identify the informative content of the proposition p or not- p as trivial, as it does not exclude any world from the logical space. However, the latter proposition has an inquisitive content which is not trivial: it raises the *issue* whether p holds. We will see that identifying propositions with sets of sets of possible worlds can uniformly accommodate these two kinds of contents.

Standard epistemic logic allowed us to represent the information state of an agent with the set $\sigma(w)$ for each w . Continuing the above inquisitive line of thinking, we may wish to represent not only what information the epistemic agent has, but also his inquisitive state, the issues the agent entertains at each possible world. Talking informally, the issue the agent entertains can be thought of a set of possibilities, i.e. information states, each of which has the potential to resolve the issue. If the agent entertains the issue whether p , then there are two possibilities to resolve the issue, either by determining that p or that not- p . Hence, thinking more formally, we think of issues as non-empty sets (as something has to resolve the issue), which consist of sets of possible worlds. Now, if a set s can resolve an issue, it means that it contains a certain sufficient amount of information; in that case, any subset of s should also resolve the issue, as any subset of s contains at least as much information as s . In other words, issues should be *downward closed*: if the set s is a member of an issue I , $s \in I$, then for any t , s.t. $t \subseteq s$, $t \in I$.

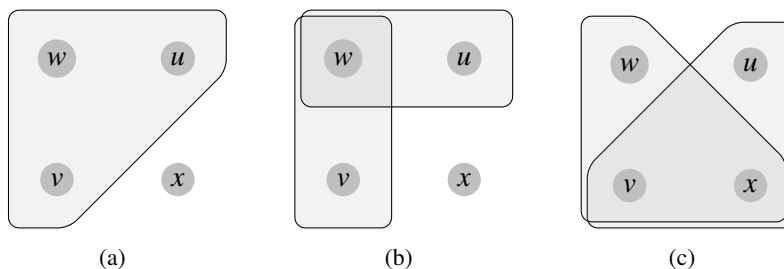


Figure 1: Examples of different issues over the same logical space

In Figure 1, a few examples of different issues are given, over the same logical space. The full line shaded areas denote information states as *elements* of an issue (the downward closure requirement was omitted from the pictures for reasons of graphical simplicity; we follow this convention in all figures to follow).

Issues should be thought of as asking the question “am I in this region of the logical space or that region?” Thus, in (b) the issue is whether we are in the region $\{w, u\}$, or the region $\{w, v\}$. Intuitively, we resolve an issue by answering this question. Note that issues can be trivial in the sense that they only present us with one option to choose from. This is the case in (a).

In inquisitive epistemic logic (IEL), the issue the agent entertains at each world is denoted $\Sigma(w)$, for each world w . Thus, $\Sigma(w)$ denotes a set of sets of possible worlds. Recall that the information state of a given agent in a given world w , $\sigma(w)$, is the set of all worlds indistinguishable from w in w , and that the issue $\Sigma(w)$ is itself a set of information states. It follows that the information state of the agent at a given world must be identified with the union of all the information states she entertains as an issue; in other words, $\bigcup \Sigma(w) = \sigma(w)$. Considering again Figure 1, one can thus think of the agent’s information states as all the world in the grey areas (regardless of overlap between the grey areas; so, e.g. the information states in (a) and (b) are identical). With this, we can define inquisitive epistemic models

quite similarly to our earlier definition of standard epistemic models. The only difference is that instead of using the epistemic map σ as a primitive, we now use an issue map Σ :

- Σ is an issue map, $\Sigma : W \rightarrow P(P(W))$, taking worlds and returning issues.

As in standard epistemic logic, we require, now from the Σ function, to respect factivity:

- for any $w \in W$, $w \in \sigma(w)$ (where $\sigma(w) = \bigcup \Sigma(w)$),

and introspection, now defined over issues:

- for any $w, v \in W$, if $v \in \sigma(w)$, then $\Sigma(v) = \Sigma(w)$ [Ciardelli, 2016, p. 266].

This introspection requirement essentially demands from the agent that if w and v are indistinguishable for the agent, then the agent entertains the same issue at each world.

The language of IEL, \mathcal{L}_{IEL} , is an extension of the language of standard epistemic logic with *questions*. This means that apart from the propositional language and the K modality, in IEL we have formulas like $p \vee\!\!\!\vee q$ expressing the question “Is p or q ?” (also known as *inquisitive disjunction*). We write $? \varphi$ as a shorthand for $\varphi \vee\!\!\!\vee \neg \varphi$; the latter is read as the question “is φ the case, or not- φ ?” The $\vee\!\!\!\vee$ connective allows us to represent knowledge-wh: we read $K(\varphi \vee\!\!\!\vee \psi)$ as “the agent knows *whether* φ or ψ .” Note that this is different from $K(\varphi \vee \psi)$, which reads “the agent knows *that* φ or ψ .”

The semantics of IEL require a deviation from standard epistemic logic: to understand the meaning of a question is not to understand when it is true, rather what information is sufficient to resolve it. Therefore, in IEL a formula is evaluated recursively relative to an information state s (a set of possible worlds) and is said to be *supported* by that state when it holds in it.

We say that an atomic formula p is supported given a model \mathcal{M} and a state s , or $\mathcal{M}, s \models p$, iff p is true in *all* worlds in s , i.e. $w \in V(p)$ for all $w \in s$. The inquisitive disjunction (= question) $\varphi \vee\!\!\!\vee \psi$ is supported in a state s iff either φ is supported in s or ψ is supported in s . Note the intuitive difference from a regular disjunction: we are not requiring that for every world w in s , the disjunction is true in w , rather, that some disjunct is true everywhere in s . Formally:

$\mathcal{M}, s \models \varphi \vee\!\!\!\vee \psi$ iff $\mathcal{M}, s \models \varphi$ or $\mathcal{M}, s \models \psi$.

We similarly lift the clause for knowledge to the level of information states:

$\mathcal{M}, s \models K\varphi$ iff for any $w \in s$, $\mathcal{M}, \sigma(w) \models \varphi$

The same goes for the other connectives (see the Appendix for full details.)

From the notion of a support condition of a formula φ relative to an information state s we can retrieve the notion of the truth condition of a formula relative to a world w by letting s be $\{w\}$. For example, the truth conditions for conjunction become the familiar:

$\mathcal{M}, \{w\} \models \psi \wedge \varphi$ iff $\mathcal{M}, \{w\} \models \psi$ and $\mathcal{M}, \{w\} \models \varphi$.

When talking about truth conditions, instead of writing the singleton $\{w\}$, we just write w . The novelty of inquisitive semantics is that one cannot in general retrieve the support conditions of a formula given its truth conditions. Note, for instance,

that relative to any world w and model \mathcal{M} , $\mathcal{M}, w \models p \vee \neg p$ is the case. However, if p is true at w and false in v , and $s = \{w, v\}$, then $\mathcal{M}, s \not\models p \vee \neg p$ (as neither p nor $\neg p$ is true both in w and in v). In that case, we say that the information state s does not resolve the question whether p .

Roughly speaking, when a sentence α does not contain the \vee operator, we call it a *declarative* sentence. Otherwise, we call it an *interrogative* sentence. For any sentence φ of the language of inquisitive epistemic logic, we have its *declarative variant*, $\varphi^!$, obtained by replacing all the \vee in φ with the regular disjunction \vee . Stated recursively, the declarative variant $\varphi^!$ of φ is:

- $\varphi^! = \varphi$ iff φ is an atom or a formula of the form $K\psi$,
- $(\varphi \wedge \psi)^! = \varphi^! \wedge \psi^!$
- $(\varphi \rightarrow \psi)^! = \varphi^! \rightarrow \psi^!$
- $(\varphi \vee \psi)^! = \varphi^! \vee \psi^!$

Next, we consider the notion of a *presupposition* of a question. When I ask whether p or q , I presuppose that the disjunction p or q is true. Accordingly, for any interrogative μ one can associate a sentence π_μ , the presupposition of μ , which shares truth conditions (but not necessarily support). In fact, the presupposition π_μ of a sentence in IEL is nothing more than the declarative variant $\mu^!$ of that sentence.

The idea behind an announcement of a sentence φ in IEL is very similar to that in epistemic logic: an announcement of φ has the effect of eliminating all the not- φ worlds from the model. The novelty of inquisitive dynamic epistemic logic is that questions can be announced, and that likewise, announcements can change the issue the agent is entertaining.

Formally, an inquisitive epistemic model \mathcal{M} after the announcement that φ , $\mathcal{M}_\varphi = (W_\varphi, V_\varphi, \Sigma_\varphi)$ is defined as:

$$W_\varphi = W \cap \{w : \mathcal{M}, w \models \varphi\}$$

$$V_\varphi = V \text{ restricted to } W_\varphi$$

$$\Sigma_\varphi = \Sigma \cap \{s : \mathcal{M}, s \models \varphi\}.$$

Note that the restriction of W is based on the truth of φ , while the restriction of Σ is based on the support conditions of φ . When considering declarative sentences, the behavior of the update is the same as an update in epistemic logic. When a question is announced, the worlds that are eliminated from the model are those in which the presupposition of the question does not hold.

Finally, we extend the IEL language to accommodate for announcements: we do so by adding the following clause to the inductive definition of the language

\mathcal{L}_{IEL} :

- If $\varphi, \psi \in \mathcal{L}_{IEL}$, then $\langle \varphi \rangle \psi \in \mathcal{L}_{IEL}$.

For a support clause, we require that ⁷

$$\mathcal{M}, s \models \langle \varphi \rangle \psi \Leftrightarrow \mathcal{M}, s \models \varphi^! \text{ and } \mathcal{M}_\varphi, s \cap \{w : \mathcal{M}, w \models \varphi\} \models \psi$$

The above clause says that $\langle \varphi \rangle \psi$ is supported wherever the declarative variant of the announcement φ is true everywhere in s and in the restricted model ψ is supported. Note that the right-hand side requirement that $\mathcal{M}, s \models \varphi^!$ guarantees that a question φ can only be successfully announced if its presupposition $\varphi^!$ is true. This concludes the survey of IEL and its dynamics (for more details about IDEL, see [Ciardelli, 2016], [Ciardelli and Roelofsen, 2015] and the Appendix).

3 Applying IDEL to contrastivism

In this section I argue that the IDEL construction $\langle \mu \rangle K$ can be understood as representing knowledge relative to μ , a context-determining question. Therefore, IDEL formulas of the form $\langle \mu \rangle K_a \varphi$ can represent Schaffer’s three-place contrastive knowledge, where:

- μ represents the contrast-question,
- a represents the agent,
- φ represents the proposition known.

Thus, instead of reading $\langle \mu \rangle \psi$ as “after announcing μ , φ holds”, we will read $\langle \mu \rangle \psi$ as “given μ , ψ holds.”

Consider again the goldfinch example. Recall that it is intuitively plausible to judge sentence (1) as true while judging sentence (2) to be false. Let us denote the sentence “There is a goldfinch in the garden” with the proposition variable gg . Similarly, we denote:

rg (raven in the garden)

cg (canary in the garden)

gn (goldfinch at the neighbors).

Alice’s initial inquisitive epistemic model looks like this:

⁷Here I am slightly diverging from the formulation of IDEL in [Ciardelli, 2016]. There, the basic dynamic modality appears in the box version as $[\varphi]\psi$. The semantic clause is

$$\mathcal{M}, s \models [\varphi]\psi \Leftrightarrow \mathcal{M}_\varphi, s \cap \{w : \mathcal{M}, w \models \varphi\} \models \psi$$

[Ciardelli, 2016, p.313]. The truth condition of $[\varphi]\psi$ relative to a world can then be retrieved to the usual

$$\mathcal{M}, w \models [\varphi]\psi \Leftrightarrow \mathcal{M}, w \models \varphi \text{ implies } \mathcal{M}_\varphi, w \models \psi$$

Moreover, the reduction axioms for the dynamic modality (see later section) are given in terms of $[\varphi]\psi$. In accordance to the general DEL validity $\langle \varphi \rangle \psi \Leftrightarrow [\varphi]\psi \wedge \varphi$, I define $\langle \varphi \rangle \psi$ in IDEL as $[\varphi]\psi \wedge \varphi^!$. This explains my choice for the support clause I give for $\langle \varphi \rangle \psi$. The intuitive difference between $\langle \varphi \rangle \psi$ and $[\varphi]\psi$ from DEL persists in IDEL: whenever φ is unsupported, $\langle \varphi \rangle \psi$ is also unsupported while $[\varphi]\psi$ is vacuously supported (as the intersection is empty). Relative to worlds, we get the usual truth conditions: whenever φ is false $\langle \varphi \rangle \psi$ is false while $[\varphi]\psi$ is vacuously true.

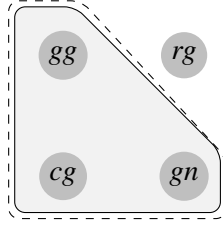


Figure 2: The initial inquisitive epistemic model for the goldfinch example.

In the initial model, Alice’s information state ($\sigma(gg)$, indicated with dashed lines) only excludes the raven possibility (from the perspective of the actual world gg). Moreover, Alice’s inquisitive state ($\Sigma(gg)$, indicated with the full line) is trivial, i.e. she is not entertaining any non-trivial issue.

We represent the effect of the question “Is it a goldfinch, or a raven?” as the following transition:

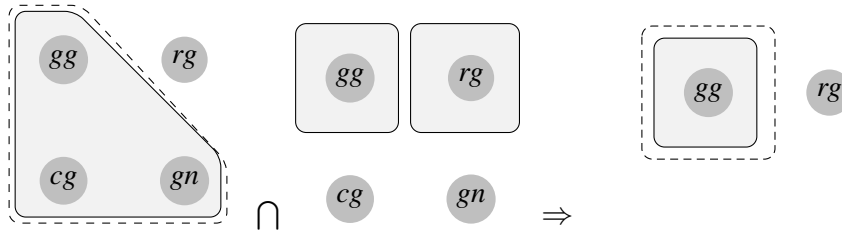


Figure 3: The effect of the question “Is it a goldfinch, or a raven?”

The left model in Figure 5 is Alice’s initial epistemic model; the middle model represents an inquisitive proposition, the question “Is it a goldfinch or a raven?”; the rightmost model is the result of intersecting the initial model with the given question, the updated model. One can check that in the right model, Alice knows that there is a goldfinch in the garden, and so she knows whether there is a goldfinch in the garden. Formally put, the sentence $\langle gg \vee rg \rangle K_a(gg \vee rg)$ is true in the actual world (the upper left world). $\langle gg \vee rg \rangle K_a(gg \vee rg)$ corresponds to sentence (1), and it has the contrastive form: a place for a question $gg \vee rg$, an agent a , and a proposition known (as a true answer to the question) $gg \vee rg$.

Next, consider the effect of updating the initial epistemic model with the question “Is it a goldfinch, or a canary?” We get the following model transition

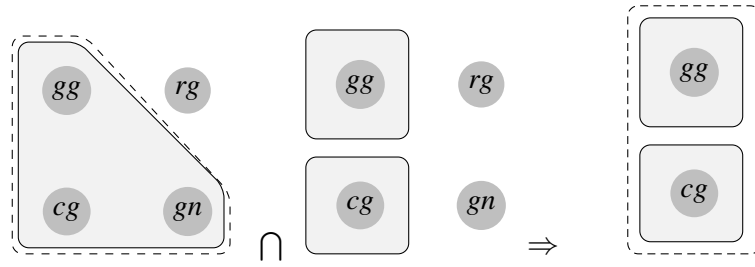


Figure 4: The effect of the question ‘Is it a goldfinch, or a canary?’

Here, the proposition depicted in the middle model represents the question “Is it a goldfinch, or a canary?”; the rightmost model of figure 6 is the updated model. In that model, it is not the case that Alice knows that it is a goldfinch in the garden, nor it is the case that she knows it is a canary. She doesn’t know whether it’s a goldfinch or a canary in the garden. In our logical language we have that $\langle gg \vee cg \rangle \neg K_a(gg \vee cg)$ is true in the actual world. This result is in line with our informal judgment that sentence (4) is false.

The problem of convergent knowledge, and Schaffer’s solution to it, can now be stated as follows: although the questions $gg \vee cg$ and $gg \vee rg$ have the same answer, namely gg , it is not the case that knowing $gg \vee cg$ is the same as knowing $gg \vee rg$. In other words, it is wrong to think about our knowledge relation as the binary $K_a(\varphi)$, as then $K_a(gg \vee rg)$ will be equivalent to $K_a(gg \vee cg)$.⁸ Pre-theoretically, the two sentences are different. Schaffer explains the difference by focusing on the different contrasts the different questions assume. In IDEL, this assumption is understood as an *update* on the initial epistemic model. With the addition of the contrastive update, we get, as needed, that $\langle gg \vee rg \rangle K_a(gg \vee rg)$ is not equivalent to $\langle gg \vee cg \rangle K_a(gg \vee cg)$ (as figures 3 and 4 show). The former knowledge ascription assumes a contrast between gg and rg ; the latter between gg and cg – the two dictate different updates. Thus, we conclude that our everyday conception of knowledge requires a place holder for a contrast-basing question, which makes it a three-place relation, rather than a two place one. Our conception of knowledge is not $K_a(\varphi)$, its $\langle \mu \rangle K_a(\varphi)$, where μ determines the contrast.

Similar formulations work for knowledge-that as well: sentence (3) and (4) from the introduction translate into $\langle gg \vee rg \rangle K_a(gg)$ and $\langle gg \vee cg \rangle K_a(gg)$, respectively. It is straightforward to check, given Figures 3 and 4, that the former is true while the latter false on the initial inquisitive model. Parallel models can be constructed for Schaffer’s other motivating examples, as they are all structurally quite similar.⁹

⁸The equivalence of the two can be verified in IDEL.

⁹E.g. the Bush example from [Schaffer, 2007b] and the diamonds example from [Schaffer and Szabo, 2014].

4 Contrastive knowledge and reduction

I return now to the issue of the reduction of contrastive knowledge to binary knowledge. The formal upshot of this section is that IDEL contrastive knowledge is not **R1** reducible to binary knowledge but is **R2** reducible.

Recall that **R1** reduction of contrastive knowledge to binary knowledge means that every ascription of contrastive knowledge of the form $K(a, \varphi, Q)$ is translatable to binary knowledge of the form $K(a, \varphi^*)$. In IDEL, **R1** reduction means that we can translate any $\langle \mu \rangle K_a \varphi$ into an equivalent sentence of the form $K_a \varphi^*$. This, however, is false.

Fact 1. *IDEL contrastive knowledge is not **R1** reducible to non-contrastive knowledge: it is not the case that for every $\langle \mu \rangle K_a \varphi$, there is a $K_a \varphi^*$ s.t.*

$\vdash \langle \mu \rangle K_a \varphi \leftrightarrow K_a \varphi^*$

Proof. (Appendix). □

Understanding the deeper relations between IDEL contrastive knowledge formulas like $\langle \mu \rangle K \varphi$ and binary knowledge formulas like $K \varphi$ will benefit from a detour via the axiomatization of IDEL. The axioms of IDEL are the combination of the axioms of static inquisitive epistemic logic (IEL) plus a set of *reduction axioms* for the update modality. The set of reduction axioms is defined for the update modality $[\varphi] \psi$. The relation between the latter modality and the one I have been using is expressed in the validity $[\varphi] \psi \wedge \varphi^! \leftrightarrow \langle \varphi \rangle \psi$. The reduction axioms are the following [Ciardelli, 2016]:

- | | |
|--|-----------------------|
| (6) $[\varphi] p \leftrightarrow (\varphi \rightarrow p)$ | atomic reduction |
| (7) $[\varphi] \perp \leftrightarrow \neg \varphi$ | falsum reduction |
| (8) $[\varphi](\chi \wedge \psi) \leftrightarrow ([\varphi]\chi \wedge [\varphi]\psi)$ | conjunction reduction |
| (9) $[\varphi](\psi \rightarrow \chi) \leftrightarrow ([\varphi]\psi \rightarrow [\varphi]\chi)$ | implication reduction |
| (10) $[\varphi](\psi \vee \chi) \leftrightarrow ([\varphi]\psi \vee [\varphi]\chi)$ | question reduction |
| (11) $[\varphi] K \psi \leftrightarrow (\varphi \rightarrow K[\varphi]\psi)$ | knowledge reduction |

The above axioms are *reduction* axioms in the sense that they specify how to reduce the complexity of a formula in a scope of the update modality. Note that each expression in the scope of the update modality in the left-hand side of these equivalences is longer than the one on the right-hand side.

The set of reduction axioms offers a simple *recipe* for turning any formula of the form $\langle \varphi \rangle \psi$ into a formula φ^* without any update modality (neither $\langle \ \rangle$ nor $[\ \]$): start with changing $\langle \varphi \rangle \psi$ to $\varphi^! \wedge [\varphi]\psi$; then use one of the reduction axioms to simplify the right conjunct $[\varphi]\psi$ according to the structure of ψ . For instance, if the main connective in ψ is an implication, use axiom (9). Repeat this process

until the scope of the update modality only contains an atomic sentence; then apply axiom (6), which eliminates the update modality.

The correctness of the above algorithm in effect shows that contrastive knowledge is **R2** reducible to binary knowledge: every contrastive knowledge sentence is reducible to a sentence without contrastive knowledge.

Fact 2. *Contrastive knowledge is **R2** reducible to binary knowledge: for every $\langle \mu \rangle K_a \varphi$, there is a φ^* s.t. $\vdash \langle \mu \rangle K_a \varphi \leftrightarrow \varphi^*$, where φ^* does not contain update operators.*

Proof. By repeatedly applying the reduction axioms (6)-(11) (for more details, including the soundness of these axioms, see [Ciardelli, 2016, sec. 8.3]) \square

For an example of such reduction in IDEL, consider again sentence (4): “Alice knows that there is a goldfinch in the garden, given the question ‘Is it a goldfinch or a raven?’” We represented it as $\langle gg \vee rg \rangle K(gg)$. Here we apply the reductive recipe on the latter:

$$(12) \langle gg \vee rg \rangle K(gg) \leftrightarrow$$

$$(13) (gg \vee rg) \wedge [gg \vee rg]K(gg) \leftrightarrow \quad \text{the principle } \langle \varphi \rangle \psi \leftrightarrow \varphi^! \wedge [\varphi] \psi$$

$$(14) (gg \vee rg) \wedge (gg \vee rg \rightarrow K[gg \vee rg]gg) \leftrightarrow \quad \text{using (11)}$$

$$(15) (gg \vee rg) \wedge (gg \vee rg \rightarrow K((gg \vee rg) \rightarrow gg)) \leftrightarrow \quad \text{using (6)}$$

(15) is provably equivalent to $\langle gg \vee rg \rangle K(gg)$, and does not include any update modalities and therefore any instances of contrastive knowledge. One might worry that since (15) contains instances of the inquisitive disjunction operator \vee , including in the scope of knowledge modality, (15) fails to show that question-relative contrastive knowledge is reducible to non-question-relative knowledge. However, (15) is further equivalent to

$$(16) (gg \vee rg) \wedge K((gg \vee rg) \rightarrow gg)$$

which does not contain any instances of the inquisitive disjunction. In general, any sentence of the form $\langle \mu \rangle K\varphi$, where φ is a sentence of propositional logic, is equivalent to a sentence without any inquisitive disjunctions.¹⁰

(16) reads: the presupposition of the question “Is it a goldfinch or raven?” holds, and the agent knows that if it is either a goldfinch or a raven in the garden, then it is a goldfinch.

¹⁰The full reasons for this fact go beyond the technical scope of this paper (see [Ciardelli, 2016]). Note that sentences of the form $\langle \mu \rangle K\varphi$ are *declaratives*, and so *truth conditional* (see [Ciardelli, 2016], p. 318), meaning, roughly, that they have no inquisitive content. Furthermore, any truth conditional sentence φ of IDEL that does not contain the E operator is equivalent to a sentence without inquisitive disjunctions (ibid. p. 209).

The reduced sentence $(gg \vee rg) \wedge K((gg \vee rg) \rightarrow gg)$ says something about the agent’s binary knowledge, but *also* about the state of the world: it says that the question $gg \vee rg$ has a true answer. Since **R1** reductions *only* describes the epistemic state of the agent, we can see why it fails. Ascribing contrastive knowledge is nothing more than ascribing binary knowledge plus giving some description of the non-epistemic state of the world.

The reduction of contrastive knowledge to binary knowledge reveals the following: in all of his examples, Schaffer is using questions which are *informative*, in the sense that the presupposition of the contrast-basing question is *not* trivial. Example: the question “is it a goldfinch or a raven?” is informative in the sense that it tells us that the bird in the garden must be either a goldfinch or a raven (but not a canary). When we ascribe contrastive knowledge, we are assuming that this informative presupposition is given to the agent. In terms of IDEL, the contrastivist’s updates always eliminate worlds from the initial model. With contrastive knowledge, we say something like “given that the agent knows the presupposition of the contrastive question, the agent knows p .”

One might wonder why the **R2** reduction I presented goes against Schaffer’s philosophical project. After all, I am showing that the three-place contrastive knowledge ascriptions of Schaffer are translatable into sentences that include binary knowledge *and* information about a presupposition of a question. In that case, my translation is also essentially a three-place relation: it requires an agent, a proposition known, and some informative presupposition. In other words, knowledge still takes three arguments, as Schaffer argues.

I agree that contrastive knowledge ascriptions require three arguments. The point of the translation is to show that there is no need to conclude that the knowledge relation itself is a three-place relation. To see why, consider the following artificial toy example. Take the relation $Bo(X)$ to stand for belief on day X of the week. We can say, for example, “ $a boT(Tue)$ that p ” to mean that a believes on a Tuesday that p . Now, any $Bo(X)$ ascription necessarily requires three arguments: a believing agent a , a proposition believed p , and a day of the week. Should we conclude that there is an irreducible state of reality beyond and above the state of belief which corresponds to $Bo(X)$? Of course not, since a very simple translation can take any sentence with $Bo(X)$ and replace with a sentence without that relation. $Bo(X)$ doesn’t add anything to the expressive power of our language, and clearly there is no reason to conclude that it picks out some state of reality that our regular concepts could not have picked. From the fact that the sentence “ $a boT(Tue)$ that p ” requires three arguments we do not conclude that such a sentence describes some novel and irreducible state of reality; from the fact that the sentence “ a knows that p , given Q ” requires three arguments we should not conclude that such a sentence describes some novel and irreducible three-place knowledge relation.

Recall that Schaffer has argued that since contrastivism best explains natural language knowledge ascriptions, and since there is no straightforward way to reduce contrastive knowledge to binary knowledge, the knowledge relation itself is contrastive. The IDEL translation blocks Schaffer’s argument. There is a way

to reduce sentences with contrastive knowledge into truth functionally equivalent sentences without contrastive knowledge. This does *not* mean that contrastivism is not a good semantic theory regarding natural language knowledge ascriptions. However, it does mean that there is no reason to conclude, as Schaffer does, that *the knowledge relation itself is contrastive*. Knowledge ascription in natural language might very well depend on a deeper contrastive structure. But the translation shows that the contrastive relation itself is based on a more basic binary conception of the knowledge relation. Schaffer's leap from the semantics of knowledge to its metaphysics is unjustified.

Since everything that can be said about knowledge can be said with binary knowledge, the skeptics are entitled to formulate their worries with binary knowledge. The contrastivist argument against the skeptic fails.

Schaffer argues that a major weakness of the binary approaches to the contrastive phenomena is that they are *linguistically implausible* [Schaffer, 2008, p. 237]. Briefly considering what he calls the conditional approach, he concludes

The thought that contrastive constructions can be analyzed via conditionals seems to be a pure invention, fabricated solely to fit the knowledge ascription data onto the Procrustean bed of Ksp [the binary conception]. (p. 237)

The IDEL translation I present here seems to be open to an objection from linguistic implausibility (witness the “implausible” complexity of (16)). However, I do not think it poses a serious threat to my argument. First, one can still accept that the contrastive knowledge relation (i.e. $\langle \mu \rangle K_a \varphi$) is the best linguistic explanation to the semantics of knowledge ascriptions. The point is that since the contrastive relation itself can be further analyzed away, there is no need to conclude that knowledge itself is contrastive. I don't see any problem with that claim; as a matter of fact, it is quite predictable that the further we analyze a linguistic object, the further we get from the surface structure of that object.

Second, it is *false* that the IDEL translation I present here is “a pure invention, fabricated in order to fit the data onto the binary conception of knowledge.” The reduction axioms of IDEL have nothing to do with Schaffer's motivations for contrastivism. Translation via reduction axioms is definitely not an ad hoc tool created to save a binary theorist of knowledge – it is a standard part of contemporary epistemic logic (see, e.g. [Baltag and Renne, 2016]).

I conclude with the following dilemma: either our contrastivist accepts the IDEL representation presented here and with it the reduction of contrastive constructions to binary ones, or the contrastivist should admit that Schaffer's examples do not really support epistemic contrastivism. As a response to the latter option, we might consider motivating and developing epistemic contrastivism beyond the simple examples discussed here. In any cases, I believe, progress in the understanding of contrastivism will be achieved.

5 Appendix

5.1 Inquisitive dynamic epistemic logic

Definition 1. *Issue*

An issue I is a non-empty, downward closed set of information states. If $t \in I$ then t is said to resolve I .

With the notion of an issue one can define an inquisitive epistemic model.

Definition 2. *inquisitive epistemic model*

An inquisitive epistemic model is a triple $\mathcal{M} = (W, V, \Sigma)$ s.t.

- W is a set of possible worlds,
- V is a valuation function, $V : \mathcal{P} \rightarrow P(W)$, and
- Σ is a state map, $\Sigma : W \rightarrow P(P(W))$, taking worlds and returning issues.

Definition 3. *the language of IDEL*

The entire language of IDEL is defined inductively as:

$$\varphi := p \mid \perp \mid \varphi \rightarrow \psi \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid K\varphi \mid E\varphi \mid [[\varphi]]\psi$$

We abbreviate $\varphi \rightarrow \perp$ as $\neg\varphi$, $\neg(\neg\varphi \wedge \neg\psi)$ as $\varphi \vee \psi$, $\alpha \vee \neg\alpha$ as $? \alpha$, and $\langle \varphi \rangle \psi$ as $\varphi^! \wedge [[\varphi]]\psi$.

Definition 4. *support*

Let \mathcal{M} be an inquisitive epistemic model and s an information state in \mathcal{M} .

Then:

1. $\mathcal{M}, s \models p$ iff $w \in V(p)$ for all $w \in s$
2. $\mathcal{M}, s \models \perp$ iff $s = \emptyset$
3. $\mathcal{M}, s \models \varphi \vee \psi$ iff $\mathcal{M}, s \models \varphi$ or $\mathcal{M}, s \models \psi$
4. $\mathcal{M}, s \models \varphi \wedge \psi$ iff $\mathcal{M}, s \models \varphi$ and $\mathcal{M}, s \models \psi$
5. $\mathcal{M}, s \models \alpha \rightarrow \varphi$ iff for any $t \subseteq s$, if $\mathcal{M}, t \models \alpha$, then $\mathcal{M}, t \models \varphi$
6. $\mathcal{M}, s \models K\varphi$ iff for any $w \in s$, $\mathcal{M}, \sigma(w) \models \varphi$
7. $\mathcal{M}, s \models E\varphi$ iff for any $w \in s$ and for any $t \in \Sigma(w)$, $\mathcal{M}, t \models \varphi$
8. $\mathcal{M}, s \models \langle \varphi \rangle \psi$ iff $\mathcal{M}, s \models \varphi^!$ and $\mathcal{M}_\varphi, s \cap \{w : \mathcal{M}, w \models \varphi\} \models \psi$

For the derived connectives the following support conditions follow:

$\mathcal{M}, s \models \neg\alpha$ iff for any non-empty $t, t \subseteq s$, $\mathcal{M}, t \not\models \alpha$

$\mathcal{M}, s \models \alpha \vee \beta$ iff there are t_1, t_2 s.t. $s = t_1 \cup t_2$, $\mathcal{M}, t_1 \models \alpha$ and $\mathcal{M}, t_2 \models \beta$.

To understand clause 8., we need a definition of an updated model:

Definition 5. *Updated model*

an inquisitive epistemic model \mathcal{M} after the announcement that φ , $\mathcal{M}_\varphi = (W_\varphi, V_\varphi, \Sigma_\varphi)$ is defined as:

$$\begin{aligned}
W_\varphi &= W \cap \{w : \mathcal{M}, w \models \varphi\} \\
V_\varphi &= V \text{ restricted to } W_\varphi \\
\Sigma_\varphi &= \Sigma \cap \{s : \mathcal{M}, s \models \varphi\}
\end{aligned}$$

Proof of Fact 1:

Proof. PAL will suffice for the proof, since IDEL is a conservative extension of PAL [Ciardelli, 2016].

Assume for reductio that Fact 1 is false, and consider the sentence $\langle p \rangle Kp$. Thus, there is a sentence $K\varphi^*$ s.t. $\vdash \langle p \rangle Kp \leftrightarrow K\varphi^*$. By the reduction axioms of PAL, we have that $\langle p \rangle Kp$ is provably equivalent to p , so $\vdash p \leftrightarrow K\varphi^*$. Consider an S5 model containing only w and v with the universal relation, and let p be true in w and false in v . Since $w \models p$, $w \models K\varphi^*$. Since the model is S5, $v \models K\varphi^*$. On the other hand, since $v \models \neg p$, we have that $v \models \neg K\varphi^*$, contradiction. \square

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